DP-003-1016001 Seat No. ________

B. Sc. (Sem. VI) (CBCS) (W.E.F. 2016) Examination

March - 2022

Graph Theory & Complex Analysis - II : Paper - 08

(Old Course)

Faculty Code : 003
Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 (a) Answer the following questions : **4**

(1) State the second theorem of graph theory.

(2) Define union of two graph.

(3) Define unicursal graph.

(4) Define null graph.

(b) Attempt any **one** : **2**

(1) Prove complete graph Kn has $\frac{n(n-1)}{2}$ edges.

(2) State and prove first theorem of graph theory.

(c) Attempt any **one** : **3**

(1) Obtain the number of internal vertices in a binary tree with n-vertices.

(2) If any graph has exactly two vertices of odd degree than there must be a path joining these two vertices.

(d) Attempt any **one** : **5**

(1) State and prove the necessary and sufficient condition for a graph G to be disconnected.

(2) Prove the tree T with n-vertices has $n-1$ edges.

2 (a) Answer the following questions : **4**

(1) Define adjacency matrix.

(2) Define vertex connectivity of graph.

(3) What is the chromatic number of complete graph Kn ?

(4) Give the dim(Ws) of graph G.

(b) Attempt any **one** : **2**

(1) Give the process to find decyclization of graph G.

(2) Prove every tree with two or more vertices is 2-chromatic.

(c) Attempt any **one** : **3**

(1) Prove $(WT, \oplus, \bullet)$ is a subspace of WG over the field GF_2.

(2) Define path matrix and state its property.

(d) Attempt any **one** : **5**

(1) If G is a (n, e) graph with f-faces and k-components then $n - e + f = k + 1$.

(2) Define minimal covering. Prove a covering g of a graph is minimal if and only if g contains no paths of length three or more.

3 (a) Answer the following questions : **4**

(1) Define mapping.

(2) Define rotation mapping.

(3) Define fixed point of transformation $W = \frac{1}{2}$.

(4) Define conformal mapping.

(b) Attempt any **one** : **2**

(1) Discuss the critical points of bilinear map.

(2) Find the fixed points of $W = \frac{z-1}{z+1}$.

(c) Attempt any **one** : **3**

(1) Obtain a transformation of sector $r \leq 1, 0 \leq \theta \leq \frac{\pi}{2}$ under the mapping $W = z^2$.

(2) Find the mobious mapping which transforms points $z = -1, \infty, 1$ of z-plane into $W = 2, 1, 0$ of W-plane.

(d) Attempt any **one** : **5**

(1) Discuss the bilinear mapping $W = \frac{1}{z}$.

(2) Show that the composition of two bilinear maps is again bilinear map.

4 (a) Answer the following questions : 4

(1) Write expansion of $\sinh z$ in Maclaurin's series.

(2) Define absolute convergence series.

(3) Find the radius of convergence of $\sum_{1}^{\infty} \frac{z^n}{2^n + 1}$.

(4) Define complex series.

(b) Attempt any **one** : 2

(1) Expand $\frac{1}{z}$ in power of $z-1$.

(2) Prove that $\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (n+1) z^n$.

(c) Attempt any **one** : 3

(1) In usual notation prove that

$$\sinh\left(z - z^{-1}\right) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n - z^{-n}\right).$$

(2) If the series $\sum z_n$ is absolute convergent then $\sum z_n$ is also convergent.

(d) Attempt any **one** : 5

(1) State and prove Laurentz's infinite series for an analytic function $f(z)$.

(2) Expand $\frac{1}{(z-1)(z-2)}$ on Laurent's series for (1) $|z-1| < 1$ (2) $|z-2| < 1$.

5 (a) Answer the following questions : 4

(1) Define singular point.

(2) Write the principle part of Laurent's expansion.

(3) Find $\text{Res}\left(\frac{\cos z}{z}, 0\right)$.

(4) Find singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$.

(b) Attempt any **one** : **2**

(1) Prove $\int_{0}^{\infty} \frac{dx}{x^2+1} = \frac{\pi}{2}$.

(2) Evaluate $\int_{C} \frac{(3z^2+2)dz}{(z-1)(z^2+9)}$ where $C', |z| = 2$.

(c) Attempt any **one** : **3**

(1) Derive the formula for finding residue of $f(z)$ at simple pole z_0.

(2) Prove $\int_{0}^{\infty} \frac{\cos x dx}{x^2+1} = \frac{\pi}{2e}$.

(d) Attempt any **one** : **5**

(1) State and prove Cauchy's residue theorem.

(2) Prove $\int_{0}^{2\pi} \frac{\cos 3\theta d\theta}{5-4\cos\theta} = \frac{\pi}{12}$.